Step 6. Compute the t-test statistic, t.

$$t=\frac{|\bar{X}_c-\bar{X}_a|}{\sqrt{\frac{s_p^2}{n_c}+\frac{s_p^2}{n_a}}}$$

$$t=\frac{|6.15-5.64|}{\sqrt{\frac{0.067}{21}+\frac{0.067}{8}}}=\frac{0.51}{\sqrt{0.0116}}=4.735$$

Step 7. Determine the critical t value, t_{crit}, for the pooled degrees of freedom.

degrees of freedom $= (n_c+n_a-2)=(21+8-2)=27.$

From Table 2, for $\alpha=0.01$ and 27 degrees of freedom, $t_{crit} = 2.771$.

Conclusion: Since 4.735 > 2.771, we assume that the sample means are not equal. It is therefore probable that the two sets of tests did not come from the same population.

Example Problem - Case 2:

A contractor has run 25 QC tests and the SHA has run 10 acceptance tests over the same period of time for the same material property. The results are shown below. Is it likely that the test came from the same population?

QC Test Results	Acceptance Test Results
21.4	34.7
20.2	16.8
24.5	16.2
24.2	27.7
23.1	20.3
22.7	16.8
23.5	20.0
15.5	19.0
17.9	11.3
24.1	22.3
18.6	
15.9	
17.0	
20.0	
24.2	
14.6	
19.7	
16.0	
23.1	
20.8	
14.6	
16.4	
22.0	
18.7	
24.2	

First, use the F-test to determine whether or not to assume the variances of the QC tests differ from the acceptance tests.

Step 1. Compute the variance, S^2, for each set of tests.

$$S_c^2 = 11.50 \qquad S_a^2 = 43.30$$

Step 2. Compute F, using the largest S^2 in the numerator.

$$F = \frac{S_a^2}{S_c^2} = \frac{43.30}{11.50} = 3.76$$

Step 3. Determine F_{crit} from Table 1 being sure to use the correct degrees of freedom for the numerator (n_a-1=10-1=9) and the denominator (n_c-1=25-1=24). From Table 1, F_{crit} = 3.69.

Conclusion: Since $F > F_{crit}$ (i.e., 3.76 > 3.69), there is reason to believe that the two sets of tests have different variabilities. That is, it is likely that they came from populations with different variances. Since we assume that the variances are not equal, we use the individual sample variances to calculate the t-test statistic, and the approximate degrees of freedom to determine the critical t value, t_{crit}.

Step 4. Compute the mean, $\bar{X}$, for each set of tests.

$$\bar{X}_c = 20.1 \qquad \bar{X}_a = 20.5$$

Step 5. Compute the t-test statistic, t.

$$t = \frac{|\bar{X}_c - \bar{X}_a|}{\sqrt{\frac{S_c^2}{n_c} + \frac{S_a^2}{n_a}}}$$

$$t = \frac{|20.5 - 20.1|}{\sqrt{\frac{11.50}{25} + \frac{43.30}{10}}} = \frac{0.4}{\sqrt{4.79}} = 0.183$$

Step 6. Determine the critical t value, t_{crit}, for the approximate degrees of freedom, f'. Remember that the calculated effective degrees of freedom is rounded down to a whole number.

$$f' = \frac{\left(\frac{s_c^2}{n_c} + \frac{s_a^2}{n_a}\right)^2}{\left(\frac{\left(\frac{s_c^2}{n_c}\right)^2}{n_c+1} + \frac{\left(\frac{s_a^2}{n_a}\right)^2}{n_a+1}\right)} - 2$$

$$f' = \frac{\left(\frac{11.50}{25} + \frac{43.30}{10}\right)^2}{\left(\frac{\left(\frac{11.50}{25}\right)^2}{26} + \frac{\left(\frac{43.30}{10}\right)^2}{11}\right)^2} - 2 = \frac{(4.79)^2}{1.713} - 2 = 11$$

From Table 2, for $\alpha = 0.01$ and 11 degrees of freedom, $t_{crit} = 3.106$.

Conclusion: Since $t < t_{crit}$, (i.e., $0.183 < 3.106$), there is no reason to assume that the sample means are not equal. It is therefore reasonable to assume that the sets of test results came from populations that had the same mean.

Table 1. Critical Values, F_{crit}, for the F-test for a Level of Significance, $\alpha = 0.01$*.

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR	1	2	3	4	5	6	7	8	9	10	11	12
1	16200	20000	21600	22500	23100	23400	23700	23900	24100	24200	24300	24400
2	198	199	199	199	199	199	199	199	199	199	199	199
3	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.5	43.4
4	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.8	20.7
5	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.5	13.4
6	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.1	10.0
7	16.2	12.4	10.9	10.0	9.52	9.16	8.89	8.68	8.51	8.38	8.27	8.18
8	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.10	7.01
9	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.31	6.23
10	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.75	5.66
11	12.2	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54	5.42	5.32	5.24
12	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.99	4.91
15	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.33	4.25
20	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.76	3.68
24	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	3.59	3.50	3.42
30	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18
40	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95
60	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.82	2.74
120	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54
∞	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36

* NOTE: This is for a two-tailed test with the null and alternate hypotheses shown below:

$$H_o: S_c^2 = S_a^2$$

$$H_a: S_c^2 \neq S_a^2$$

Table 1. Critical Values, F_{crit}, for the F-test for a Level of Significance, $\alpha = 0.01$*. *(continued)*

DEGREES OF FREEDOM FOR NUMERATOR

DEGREES OF FREEDOM FOR DENOMINATOR	15	20	24	30	40	50	60	100	120	200	500	∞
1	24600	24800	24900	25000	25100	25200	25300	25300	25400	25400	25400	25500
2	199	199	199	199	199	199	199	199	199	199	199	200
3	43.1	42.8	42.6	42.5	42.3	42.2	42.1	42.0	42.0	41.9	41.9	41.8
4	20.4	20.2	20.0	19.9	19.8	19.7	19.6	19.5	19.5	19.4	19.4	19.3
5	13.1	12.9	12.8	12.7	12.5	12.5	12.4	12.3	12.3	12.2	12.2	12.1
6	9.81	9.59	9.47	9.36	9.24	9.17	9.12	9.03	9.00	8.95	8.91	8.88
7	7.97	7.75	7.65	7.53	7.42	7.35	7.31	7.22	7.19	7.15	7.10	7.08
8	6.81	6.61	6.50	6.40	6.29	6.22	6.18	6.09	6.06	6.02	5.98	5.95
9	6.03	5.83	5.73	5.62	5.52	5.45	5.41	5.32	5.30	5.26	5.21	5.19
10	5.47	5.27	5.17	5.07	4.97	4.90	4.86	4.77	4.75	4.71	4.67	4.64
11	5.05	4.86	4.76	4.65	4.55	4.49	4.45	4.36	4.34	4.29	4.25	4.23
12	4.72	4.53	4.43	4.33	4.23	4.17	4.12	4.04	4.01	3.97	3.93	3.90
15	4.07	3.88	3.79	3.69	3.59	3.52	3.48	3.39	3.37	3.33	3.29	3.26
20	3.50	3.32	3.22	3.12	3.02	2.96	2.92	2.83	2.81	2.76	2.72	2.69
24	3.25	3.06	2.97	2.87	2.77	2.70	2.66	2.57	2.55	2.50	2.46	2.43
30	3.01	2.82	2.73	2.63	2.52	2.46	2.42	2.32	2.30	2.25	2.21	2.18
40	2.78	2.60	2.50	2.40	2.30	2.23	2.18	2.09	2.06	2.01	1.96	1.93
60	2.57	2.39	2.29	2.19	2.08	2.01	1.96	1.86	1.83	1.78	1.73	1.69
120	2.37	2.19	2.09	1.98	1.87	1.80	1.75	1.64	1.61	1.54	1.48	1.43
∞	2.19	2.00	1.90	1.79	1.67	1.59	1.53	1.40	1.36	1.28	1.17	1.00

*** NOTE:** **This is for a two-tailed test with null and alternate hypotheses shown below:**

$$H_o: S_c^2 = S_a^2$$

$$H_a: S_c^2 \neq S_a^2$$

Table 2. Critical Values, t_{crit}, for the t-test* for Various Levels of Significance.

degrees of freedom	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.10$
1	63.657	12.706	6.314
2	9.925	4.303	2.920
3	5.841	3.182	2.353
4	4.604	2.776	2.132
5	4.032	2.571	2.015
6	3.707	2.447	1.943
7	3.499	2.365	1.895
8	3.355	2.306	1.860
9	3.250	2.262	1.833
10	3.169	2.228	1.812
11	3.106	2.201	1.796
12	3.055	2.179	1.782
13	3.012	2.160	1.771
14	2.977	2.145	1.761
15	2.947	2.131	1.753
16	2.921	2.120	1.746
17	2.898	2.110	1.740
18	2.878	2.101	1.734
19	2.861	2.093	1.729
20	2.845	2.086	1.725
21	2.831	2.080	1.721
22	2.819	2.074	1.717
23	2.807	2.069	1.714
24	2.797	2.064	1.711
25	2.787	2.060	1.708
26	2.779	2.056	1.706
27	2.771	2.052	1.703
28	2.763	2.048	1.701
29	2.756	2.045	1.699
30	2.750	2.042	1.697
40	2.704	2.021	1.684
60	2.660	2.000	1.671
120	2.617	1.980	1.658
∞	2.576	1.960	1.645

*** NOTE:** This is for a two-tailed test with the null and alternate hypotheses shown below:

$$H_0: \bar{X}_c = \bar{X}_a$$
$$H_a: \bar{X}_c \neq \bar{X}_a$$